SECTION 1

1 Integers, powers and roots

Directed numbers

Directed numbers can be positive, negative or zero. They can be added together, subtracted from each other, multiplied and divided. The rules for multiplication and division are the same:

- When both quantities are positive or both are negative, the result is positive.
- When one is positive and one is negative, the result is negative.

Exercise 1.1

Use a number line if necessary to answer questions 1–5.

1 **a)** $(-15.5) + (-9.5) =$ ________________

b) $(-17.1) - (-11.5) =$ ________________

2 **a)** $(-0.3) + (+14.3) + (-21.6) =$ ________________

b) $(+13.7) + (-22.6) + (-20.3) =$ ________________

3 **a)** $(+9.75) - (+6.3) =$ ________________

b) $(+1) - (+1.55) =$ ________________

4 **a)** $(+1.34) - (+2.56) =$ ________________

b) $(+1) - (+15.3) =$ ________________ →

5 a) $(-7.6) - (+3.17) =$ ______

b) $(-2.5) - (+2.5) =$ ______

6 Complete this multiplication grid.
Give your answers correct to two decimal places.

×	−5.2	−3.3	−1.1	0	+2.4	+3.6	+4.7
+3.2							
+2.5							
+1.4							
0							
−1.6							
−2.8							
−3.75							

7 If $xy = +8$, complete this table. (Remember, xy means x multiplied by y.)

x	+8	+4	+2	+1	−1	−2	−4	−8
y								

Powers and roots

Squaring a number is multiplying it by itself. The inverse of squaring a number is finding its **square root**. Every positive number has a positive and a negative square root. To estimate the square root of a number which is not a square number, use the square numbers that it falls between as indicators.

Example 1: $\sqrt{20}$ is between $\sqrt{16} = 4$ and $\sqrt{25} = 5$. So $\sqrt{20}$ is about ±4.5.

Cubing a number is multiplying it by itself twice. The inverse of cubing a number is finding its **cube root**. To estimate the cube root of a number which is not a cube number, use the cube numbers that it falls between as indicators.

Example 2: $\sqrt[3]{250}$ is between $\sqrt[3]{216} = 6$ and $\sqrt[3]{343} = 7$. So $\sqrt[3]{250}$ is about +6.3.

Exercise 1.2

1 Without using a calculator, work out the square roots in parts **(i)** and **(ii)**. Then estimate the square root in part **(iii)**. Give positive and negative roots.

a) (i) $\sqrt{25}$ = ____________ **(ii)** $\sqrt{36}$ = ____________

(iii) $\sqrt{30}$ ______________________________

b) (i) $\sqrt{4}$ = ____________ **(ii)** $\sqrt{9}$ = ____________

(iii) $\sqrt{5}$ ______________________________

c) (i) $\sqrt{0.81}$ = ____________ **(ii)** $\sqrt{0.64}$ = ____________

(iii) $\sqrt{0.7}$ ______________________________

d) (i) $\sqrt{0.64}$ = ____________ **(ii)** $\sqrt{0.49}$ = ____________

(iii) $\sqrt{0.55}$ ______________________________

2 Estimate each of these square roots.

a) $\sqrt{60}$ ______________________________

b) $\sqrt{77}$ ______________________________

c) $\sqrt{0.6}$ ______________________________

d) $\sqrt{0.4}$ ______________________________

3 Without using a calculator, work out the cube roots of the numbers in parts **(i)** and **(ii)**. Then estimate the cube root of the number in part **(iii)**.

a) (i) 64____________ **(ii)** 27____________

(iii) 42 ______________________________

b) (i) 125____________ **(ii)** 216____________

(iii) 180 ______________________________

c) (i) 729 ______ (ii) 512 ______

(iii) 520 ______

4 Without using a calculator, estimate each of these cube roots.

a) $\sqrt[3]{40}$ ______

b) $\sqrt[3]{300}$ ______

c) $\sqrt[3]{20000}$ ______

Indices

A short way to write squares, cubes and other **powers** is to use **index notation**.
Example 1: $3 \times 3 \times 3 \times 3 \times 3 = 3^5$

The basic **laws of indices** are:

- $a^m \times a^n = a^{(m+n)}$
- $a^m \div a^n = a^{(m-n)}$
- $(a^m)^n = a^{mn}$

Example 2: Simplify $5^3 \times 5^2 = 5^{(3+2)} = 5^5$
Example 3: Evaluate $(3^2)^3 = 3^{(2\times3)} = 3^6 = 729$
Example 4: Evaluate $6^{-4} \times 6^{12} \times 6^{-6} = 6^{(-4+12-6)} = 6^2 = 6 \times 6 = 36$

If the base numbers are not the same, only parts of the expression can be simplified using the index laws.
Example 5: $2 \times 2 \times 2 \times 7 \times 7 = 2^3 \times 7^2$

Two further laws of indices are:

- $a^0 = 1$
- $a^{-m} = \frac{1}{a^m}$

Example 6: $12^0 = 1$
Example 7: $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
Example 8: $(5^3)^4 \div (5^4)^3 = 5^{12} \div 5^{12} = 5^0 = 1$

Exercise 1.3

1 Simplify the following using indices.

a) $4 \times 4 \times 4 \times 4 =$ ______

b) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 =$ ______

2 Write out the following in full.

a) $12^4 =$ ______________________________

b) $8^5 =$ ______________________________

3 Simplify the following using indices.

a) $2^2 \times 2^3 \times 2^4 =$ ______________________________

b) $15 \times 15^2 \times 15 \times 15^2 =$ ______________________________

c) $2^2 \times 2^3 \times 2^4 \div 2^2 =$ ______________________________

d) $7^3 \times 7^2 \times 7 \times 7 \div 7^2 =$ ______________________________

e) $9^3 \div 9^2 =$ ______________________________

f) $7^5 \div 7^2 \div 7^2 =$ ______________________________

g) $5^5 \div 5^3 =$ ______________________________

h) $3^9 \div 3^9 =$ ______________________________

4 Simplify the following.

a) $(3^5)^2 =$ ______________ **b)** $(2^3)^4 =$ ______________

c) $(7^7)^2 =$ ______________ **d)** $(8^4)^4 =$ ______________

5 Simplify the following.

a) $8^3 \times 8^4 =$ ______________________________

b) $10^5 \div 10^2 =$ ______________________________

c) $7^4 \times 7^2 \div 7^3 =$ ______________________________

d) $8^8 \times 8^2 \div 8^4 =$ ______________________________ →

e) $(2^5)^2 \div 2^3 \times (2^2)^2 \div 2^3 =$ ______

f) $(3^2)^3 \div 3^3 \times (3^2)^2 \div 3 =$ ______

6 Simplify the following. Leave your answers in index form.

a) $4 \times 4 \times 3 \times 5 \times 3 \times 4 \times 5 =$ ______

b) $2 \times 3 \times 2 \times 3 \times 5 \times 5 \times 5 \times 5 =$ ______

c) $3^2 \times 5 \times 5 \times 3 =$ ______

d) $5^2 \times 5^3 \times 3^2 \times 3 \times 5^3 \times 3^2 \times 3 \times 11 =$ ______

7 Using indices, find the value of each of these.

a) $7^4 \times 7 \div 7^5 =$ ______

b) $(2^3)^2 \div 2^3 \times (2^2)^2 \div 2^7 =$ ______

8 Without using a calculator, write each of these as an integer or a fraction.

a) $3^{-3} =$ ______

b) $9^{-2} =$ ______

c) $8 \times 4^{-1} =$ ______

d) $81 \times 3^{-2} =$ ______

e) $1000 \times 10^{-2} =$ ______

f) $128 \times 2^{-3} =$ ______

g) $4 \times 2^{-3} =$ ______

h) $9 \times 3^{-4} =$ ______

i) $32 \times 2^{-5} =$ ______

j) $2^{-6} \times 2^5 =$ ______

k) $5^{-2} \times 5 =$ ______

l) $3^7 \times 3^{-5} =$ ______

Teacher comments

2 Expressions and formulae

Index notation and algebra

The **laws of indices** apply to algebra as well as numbers.

- $a^m \times a^n = a^{(m+n)}$
- $a^m \div a^n = a^{(m-n)}$
- $(a^m)^n = a^{mn}$

Example 1: $m^3 \times m^4 = m^{(3+4)} = m^7$
Example 2: $w^5 \div w^3 = w^{(5-3)} = w^2$
Example 3: $(m^2)^3 = m^{(2\times3)} = m^6$

If the base numbers are not the same, only parts of the expression can be simplified using the index laws.
Example 4: $g \times g \times g \times h \times h \times g \times g \times h \times h = g^5 \times h^4 = g^5h^4$

Any letter (or number) raised to the power of 0 is equal to 1.

- $a^0 = 1$

Example 5: $w^5 \div w^5 = w^{(5-5)} = w^0 = 1$

Exercise 2.1

1 Simplify the following using indices.

a) $a \times a \times a \times a \times a =$ ________________

b) $b \times b \times b \times b \times b \times b \times b \times b =$ ________________

c) $c \times c \times c \times c \times c \times c \times c \times c =$ ________________

2 Write out the following in full.

a) $p^5 =$ ________________

b) $q^7 =$ ________________

c) $t^6 =$ ________________

3 Simplify the following using indices.

a) $a^3 \times a^7 =$ ______

b) $b^9 \times b^5 =$ ______

c) $c^2 \times c^4 \times c^6 =$ ______

d) $e^8 \div e^5 =$ ______

e) $g^2 \div g =$ ______

4 Simplify the following.

a) $(m^3)^3 =$ ______

b) $(p^5)^4 =$ ______

c) $(m^3)^6 =$ ______

d) $(p^2)^7 =$ ______

e) $(x^4)^5 =$ ______

5 Simplify the following.

a) $a^4 \times a^9 =$ ______

b) $b^5 \div b^4 =$ ______

c) $c^4 \times c^2 \div c =$ ______

d) $(e^4)^2 \div e^5 =$ ______

e) $(m^2)^3 \div m^5 =$ ______

6 Simplify the following. Do not write multiplication signs in your answers.

a) $p \times q \times q \times r \times r \times q \times q \times r =$ ______

b) $t \times t \times t \times t \times u \times u \times t \times t \times t \times u \times u =$ ______

c) $m \times n \times n \times m \times m \times n \times n \times n =$ ______

d) $u^2 \times v \times v \times v =$ ______

e) $s^2 \times s^3 \times t^2 \times t \times s^3 \times t^2 \times t =$ ______

7 Simplify the following using indices.

a) $a^8 \div a^8 =$ ____________ **b)** $b^2 \times b^3 \div b^5 =$ ____________

c) $c^4 \times c^2 \div c^5 =$ ____________ **d)** $x^{44} \div x^{44} =$ ____________

e) $(p^4)^3 \div (p^3)^4 =$ ____________

8 Simplify the following.

a) $n^6 \times n^7 \times n \div n^6 \times n^4 =$ ____________

b) $(a^3)^4 \div a^9 =$ ____________

c) $(b^4)^3 \div (b^2)^4 =$ ____________

d) $(n^2)^3 \times (n^4)^2 \div (n^4)^3 =$ ____________

e) $u^2 \times w^3 \times u^3 \times w \times w^3 =$ ____________

Expressions

An **expression** represents a value in algebraic form. In the expression $4x - 6$, $4x$ and -6 are **terms** in the expression. Sometimes the expression can be simplified by collecting together the **like terms**.

Factorising is the opposite of expanding brackets. To factorise an expression fully, write the highest common factor (HCF) of all the terms outside the brackets. This might be a number, a letter or a combination of letters and numbers.

Example 1: $20x + 5y = 5(4x + y)$
Example 2: $a^2 + 2ab - 3a = a(a + 2b - 3)$
Example 3: $4pq + 12p^2 = 4p(q + 3p)$

Some expressions do not have a factor that is common to all the terms but can still be factorised by **grouping**.
Example 4: $2ab + 6b + 5a + 15 = 2b(a + 3) + 5(a + 3) = (a + 3)(2b + 5)$

Exercise 2.2

1 Write an expression for the total number of counters in each of these cases, using brackets where possible.

Use r to stand for the number of counters in a full box of red counters, y for a full box of yellow counters, and b and g for full boxes of blue and green counters respectively.

→

a) 2 boxes of red from which 5 counters have been removed from each box and then the remaining number has been trebled

____

b) 3 boxes of blue from which 2 counters have been removed from each box and then the remaining number has been multiplied by 5

____

c) 2 boxes of blue and 2 boxes of red from which 6 counters have been removed from each box and then the remaining number has been doubled

____

d) 3 boxes of each colour with 8 extra in each box

____

2 Write an expression for the area and perimeter of each of these rectangles. Use brackets where possible.

a)

Area = ____________

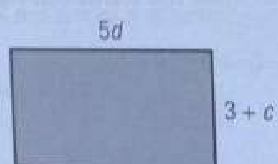
Perimeter = ____________

b)

Area = ____________

Perimeter = ____________

c)



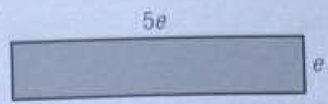
Area = ____________

Perimeter = ____________

d)

Area = ____________

Perimeter = ____________

e) 

Area = ________

Perimeter = ________

3 Factorise the following expressions fully.

a) $24p - 28q =$ ________

b) $6a - 30b =$ ________

c) $21d - 14e =$ ________

d) $6a + 9b + 12c =$ ________

e) $8a + 2b + 4c =$ ________

f) $6p + 9q + 15r =$ ________

g) $12m + 16n - 36r =$ ________

h) $7a - 14b + 35c =$ ________

i) $24p - 32q - 12r =$ ________

j) $3a - 3b - 3c =$ ________

k) $6a - 12b - 18c =$ ________

l) $7p - 7q + 7r =$ ________

m) $30p - 60q - 15r =$ ________

4 Factorise the following expressions fully.

a) $11a - 11ab =$ ________

b) $4a - 16a - 8ab =$ ________

c) $5ab - 10bc + 15b^2 =$ ________

d) $8ab^2 - 6b^2 =$ ________

e) $a^2 + a =$ ________

f) $b + b^2 =$ ________

g) $b^2 + b^3 =$ ________

h) $a^3 + a^2 + a =$ ________

i) $p^3 + 2p^2 + 3p =$ ________

j) $7m^3 - 9m^2 - 4m =$ ________

k) $56a^2b - 28ab^2 =$ ________

l) $72ab - 36bc + 48bd =$ ________

m) $4a^3b - 6a^3c =$ ________

n) $14m^3n^2 - 21m^2n =$ ________

o) $6a^2b^2 + 12ab =$ ________

p) $3c^2 - 15c^3 =$ ________

q) $5ab - 5ac =$ ________

r) $13b^2c - 26bc^2 =$ ________

→

5 Factorise the following expressions by grouping.

a) $ac - ad - bc + bd =$ ______

b) $rs - rv - ts + tv =$ ______

c) $wx - vx + vy - wy =$ ______

d) $a^2 + ac + ab + bc =$ ______

e) $pq + p^2 + pr + rq =$ ______

f) $mn + mr + n^2 + rn =$ ______

g) $px + py + rx + ry =$ ______

h) $ab - 2ac - 3bc + 6c^2 =$ ______

i) $ab - a - bd + d =$ ______

Changing the subject of a formula

A **formula** describes a relationship between different variables. The **subject** of the formula is the letter on its own on one side of the equals sign. To make a different letter the subject, rearrange the formula by doing the same to both sides.

Example: Make u the subject of the formula: $v = u + at$.

$v - at = u$ (subtract at from both sides)

Exercise 2.3

Rearrange the following formulae to make the underlined letter the subject.

1 a) $p + \underline{q} = r$

$q =$ ______

b) $\underline{q} + 2r = s$

$q =$ ______

2 a) $2q + \underline{r} = 4p$

$r =$ ______

b) $3s + \underline{q} = 2p$

$q =$ ______

3 a) $p\underline{q} = r$

$q =$ ____________

b) $p\underline{r} = qs$

$r =$ ____________

4 a) $\underline{p}q = r + 3$

$p =$ ____________

b) $p\underline{r} = q - 4$

$r =$ ____________

5 a) $m + \underline{n} = r$

$n =$ ____________

b) $m - \underline{n} = p$

$n =$ ____________

6 a) $2\underline{m} + n = 3p$

$m =$ ____________

b) $3x = 2\underline{p} + q$

$p =$ ____________

7 a) $\underline{x}y = uv$

$x =$ ____________

b) $-\underline{p}q = rs$

$p =$ ____________

8 a) $6\underline{q} = 2p - 5$

$q =$ ____________

b) $6q = 2\underline{p} - 5$

$p =$ ____________

9 a) $3x - 7y = 4\underline{z}$

$z =$ ____________

b) $3x - 7\underline{y} = 4z$

$y =$ ____________

10 a) $2p\underline{r} - q = 8$

$r =$ ____________

b) $2pr - \underline{q} = 8$

$q =$ ____________

→

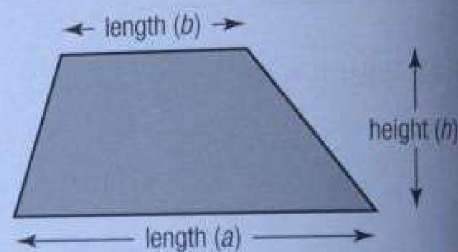
11 The distance travelled by an object is expressed in the formula $s = ut + \frac{1}{2}at^2$, where s is the distance, u is the initial (starting) velocity, a is the acceleration and t is the time.

Rearrange the formula to make:

a) u the subject **b)** a the subject.

$u =$ __________ $a =$ __________

12 The formula for calculating the area of a trapezium is $A = \frac{1}{2}(a + b)h$.



Rearrange the formula to make a the subject.

$a =$ __________

Substitution

We can **substitute** known values for the letters in an expression or formula to find an unknown value.

Example: When $a = 7$ and $b = -3$,

$a + 4b^2$

$= 7 + 4 \times (-3)^2$

$= 7 + 4 \times 9$

$= 7 + 36$

$= 43$

Exercise 2.4

1 Calculate the value of each of the following expressions when $a = \frac{1}{2}$, $b = -\frac{1}{2}$, $c = 2$ and $d = -2$. Use a calculator if necessary.

a) $a + b =$ ____________________

b) $a - b =$ ____________________

c) $a + b - c =$ ____________________

d) $a + b - c - d =$ ____________________

e) $6a =$ ____________________

f) $4b =$ ____________________

g) $6c =$ ____________________

h) $2d =$ ____________________

i) $a + b + c + d =$ ____________________

j) $a - b + c - d =$ ____________________

k) $a - b + (c - d) =$ ____________________

l) $a - b - (c - d) =$ ____________________

m) $(a - b) - (c - d) =$ ____________________

n) $a^2 =$ ____________________

o) $b^2 =$ ____________________

p) $c^2 =$ ____________________

q) $d^2 =$ ____________________

→

r) $a^2 - b^2 =$ ______

s) $a^2 - b^2 - c^2 - d^2 =$ ______

2 Given the formula $A = \frac{1}{2}(a + b)h$, find:

a) the value of A when $a = 12\,\text{cm}$, $b = 14\,\text{cm}$ and $h = 4\,\text{cm}$

______

b) the value of h when $A = 60\,\text{cm}^2$, $a = 12\,\text{cm}$ and $b = 8\,\text{cm}$

______

c) the value of b when $A = 15\,\text{cm}^2$, $a = 4\,\text{cm}$ and $h = 5\,\text{cm}$.

______

Adding and subtracting algebraic fractions

To add or subtract algebraic fractions, first change them to equivalent fractions with the same denominator if necessary.

Example: $\frac{x}{3} + \frac{y}{4} = \frac{4x}{12} + \frac{3y}{12} = \frac{4x + 3y}{12}$

Exercise 2.5

1 Simplify the following.

a) $\frac{2}{5} - \frac{1}{10} =$ ______

b) $\frac{2a}{5} - \frac{a}{10} =$ ______

c) $\frac{x}{5} + \frac{x}{10} =$ ______

d) $\frac{3a}{5} - \frac{7a}{25} =$ ______

e) $\frac{3a}{5b} - \frac{7a}{25b} =$ ____________________

f) $\frac{x}{y} - \frac{2x}{9y} =$ ____________________

g) $\frac{a}{b} + \frac{c}{3b} - \frac{d}{9b} =$ ____________________

h) $\frac{ab}{xy} + \frac{2ab}{3xy} =$ ____________________

i) $\frac{b}{5x} + \frac{b}{20x} - \frac{b}{40x} =$ ____________________

2 Simplify the following.

a) $\frac{1}{n} + \frac{2}{3n} =$ ____________________

b) $\frac{m}{n} + \frac{2m}{3n} =$ ____________________

c) $m + \frac{m}{2} =$ ____________________

d) $x + \frac{2x}{5} =$ ____________________

e) $a - \frac{3a}{8} =$ ____________________

f) $3x + \frac{x}{2} =$ ____________________

g) $8m - \frac{15m}{2} =$ ____________________

Teacher comments

3 Shapes and geometric reasoning

Polygons

A **polygon** is a two-dimensional closed shape with straight sides, for example triangles, quadrilaterals, pentagons and hexagons. The exterior angles of any polygon add up to 360°. The interior and exterior angles at any vertex add up to 180°.

The interior angles of a triangle add up to 180°.

In a **regular** polygon, all the sides are the same length and all the angles are the same size.

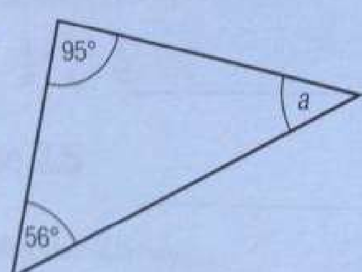
- Size of each exterior angle of a regular polygon $= \dfrac{360^\circ}{\text{number of sides}}$
- Size of each interior angle of a regular polygon $= \dfrac{\text{sum of interior angles}}{\text{number of sides}}$

When a diagram shows parallel lines, look for any pairs of **alternate** or **corresponding** angles, which are equal. (Look for 'Z' and 'F' shapes.)

Exercise 3.1

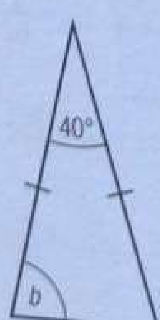
1 Calculate the size of each unknown angle in these diagrams.

a)



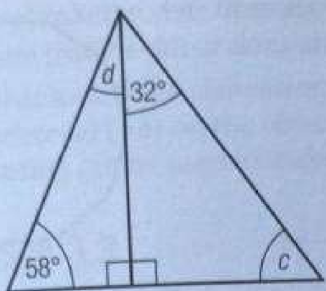
$a =$ _____

b)



$b =$ _____

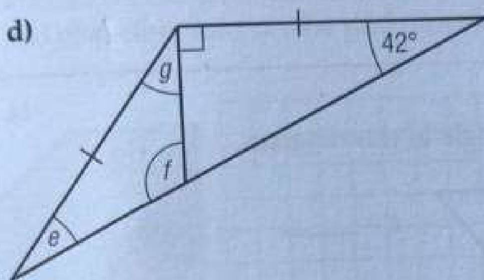
c)



$c =$ _____

$d =$ _____

d)



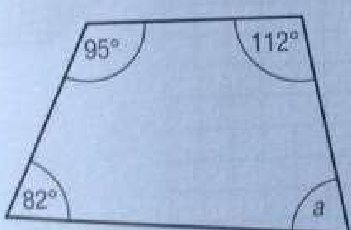
$e =$ _____

$f =$ _____

$g =$ _____

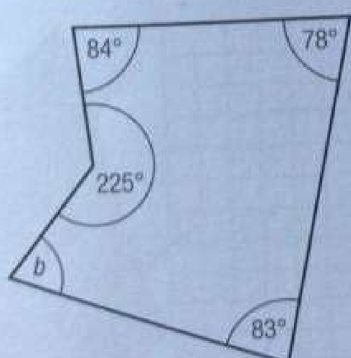
2 Calculate the size of the unknown angles in these polygons.

a)



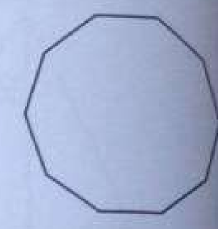
$a =$ _____

b)



$b =$ _____

3 A regular decagon has 10 sides as shown.
Calculate:

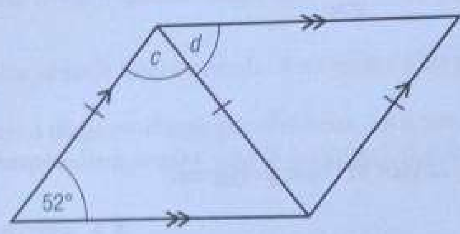


a) the size of each exterior angle ____________

b) the size of each interior angle ____________

c) the sum of all the interior angles. ____________

4 Calculate the size of each unknown angle in this diagram.



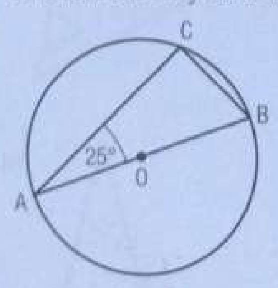
$c =$ ____

$d =$ ____

Circle geometry

Exercise 3.2

1 Calculate the size of angle B in this diagram.
Give reasons for your answer.



$B =$ ____

Drawing three-dimensional shapes

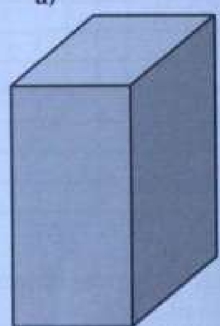
A **front elevation** is a two-dimensional view of a three-dimensional object from the front, the **side elevations** are two-dimensional views from the left and right sides, and the **plan** is the view from above. Line up the edges in the different elevations and make sure that the dimensions are consistent. This is easier if a square grid is used.

Views that look three-dimensional can be drawn by using an **isometric grid**. Vertical and horizontal lines on the object are drawn parallel to the three axes of the grid, and shading can be used to make the diagram clearer.

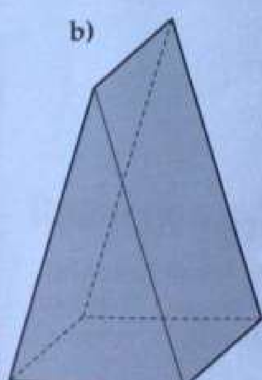
Exercise 3.3

1 For each of the three-dimensional shapes below, draw the possible front, side and plan elevations on the grid provided.

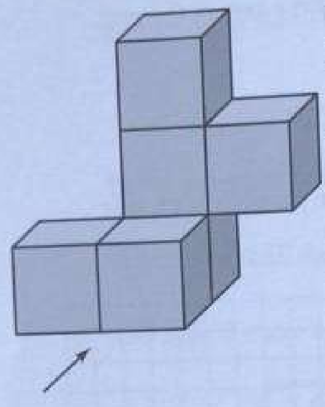
a)

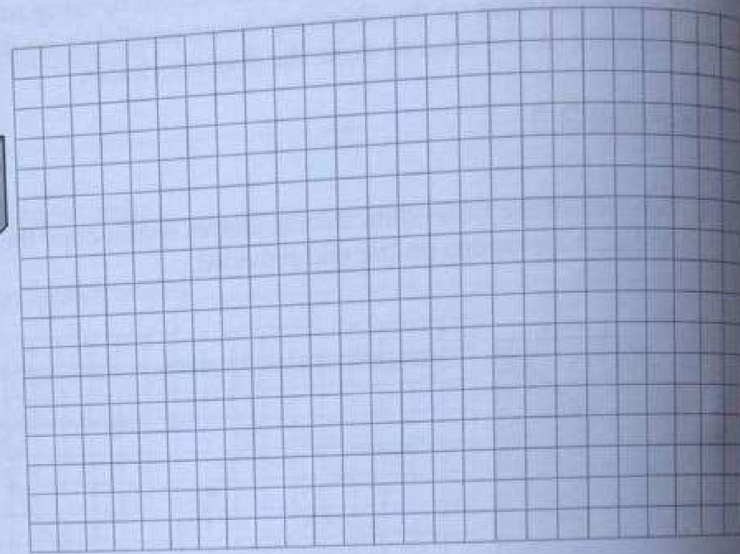


b)



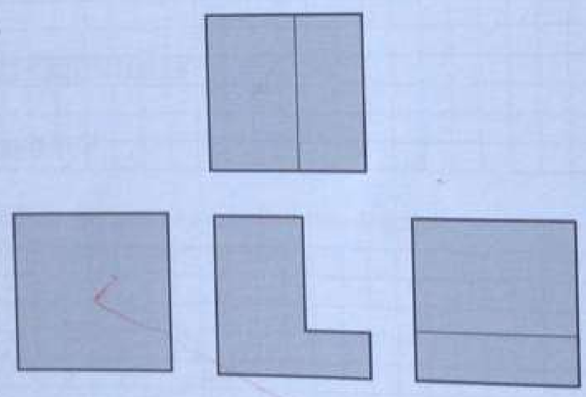
2 This diagram shows a shape made of small cubes. Draw the front, side and plan elevations on the grid provided. The arrow shows the direction of the front elevation.



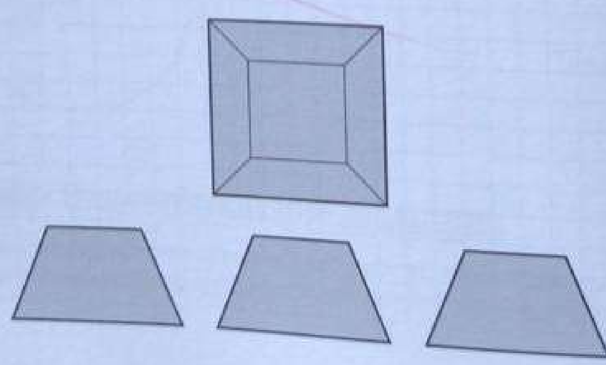


3 For each of the following sets of elevations, sketch the three-dimensional object.

a)

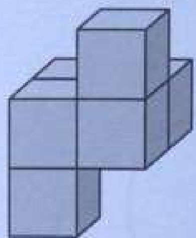


b)

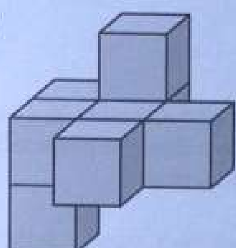


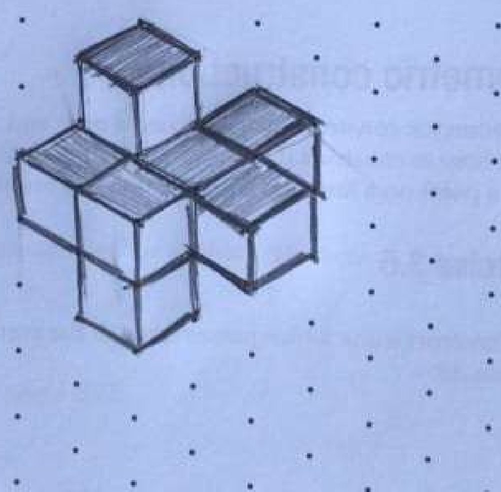
4 These diagrams show shapes made of small cubes. Make an isometric drawing of each shape on the grid provided.

a)



b)





Reflection symmetry in three-dimensional shapes

A **plane of symmetry** divides a three-dimensional shape into two congruent (identical) shapes.

Example: A cuboid has three planes of symmetry.